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AN EFFICIENT ALGORITHM TO SOLVE DYNAMIC BUDGET CONSTRAINED UNCAPACITATED FACILITY LOCATION-NETWORK DESIGN PROBLEM

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─Abstract ─
In this paper, a budget constrained dynamic (multi-period) uncapacitated facility location-network design problem (DUFLNDP) is investigated. The facility location-network design problem deals with the determination of the optimal locations of facilities and the design of the underlying network simultaneously. The objective is to minimize the total travel costs for customers and operating costs for facilities and network links subject to a constraint on the budget for opening and/or closing facilities and constructing links. We propose a mixed-integer non-linear programming model that considers a dynamic planning horizon in facility location-network design problem. An efficient hybrid algorithm based on simulated annealing algorithm and exact methods to solve the proposed model is also presented. Finally, the performance of our proposed algorithm is tested on extensive randomly generated instances and also compared with CPLEX solver.

Key Words: *facility location; network design; dynamic; hybrid algorithm*
JEL Classification: C65

1. INTRODUCTION

Facility location decisions are managerial strategic activities and are usually occurred in some important real-world applications. The general objective of facility location problems is to locate one or more facilities that service a set of demand points. Weber (1999) introduced the first paper in this field. In addition, network design problems deal with the decisions about network links construction and determining traffic flow on these links and possibly satisfies additional constraints. Usually, these decisions are made by considering the associated costs (or profits) of satisfying the demand and the costs related to establishing (or operating) the facilities and links. Essentially, the facility location-network design problem is a combination of the facility location and network design that involves the determination of the location of the facilities (as in facility location) required to satisfy a set of clients' demands and the determination of travelable links (as in network design) to connect clients to facilities.

The literature on facility location and network design theories is rich. After Weber work, many papers have been published that provide admirable introductions and reviews of the developments in the field. In addition, the network design problem is relatively newer than the facility location problem. Magnanti and Wong (1984) and Yang and Bell (1998) proposed a general problem description of the network design problem that encompasses many variations. Furthermore, they present an outstanding review of the network design problem models and discuss the solution methods.

However, the facility location-network design problem has been considered less in the literature. The uncapacitated facility location-network design problem (UFLNDP) was originally proposed by Daskin et al. (1993). Later, Melkote (1996) in his doctoral thesis investigated three models for the facility location-network design problem include: UFLNDP, the capacitated facility location-network design problem (CFLNDP), and the maximum covering location-network design problem (MCLNDP). The results of this thesis were published in (Melkote and Daskin 2001a , 2001b). Drezner and Wesolowsky (2003) proposed a new network design problem with potential links where each link can be either constructed or not at a given cost. Also, each constructed link can be constructed either as a one-way or two-way link. In this paper, four basic problems were created subject to two objective functions. Then, these problems were solved by a descent algorithm, simulated annealing, tabu search, and a genetic algorithm.

In this paper, the dynamic budget-constrained facility location-network design problem with changing the parameters such as client demand, facility and network link costs is investigated. Thus, the optimal locations of facilities and the configuration of the underlying network are determined simultaneously subject to the available budget in each time period of the planning horizon. A maximum budget for the opening and closing of facilities and construction of the network links is considered at each time period. The objective is to minimize the total traveling and operating costs over a finite time while ensuring that, at each single period, all demand is fully routed through the network. The budget-constrained DUFLNDP is clearly NP-hard because it combines two NP-hard problems. Therefore, an efficient hybrid algorithm is proposed that combines exact methods (Brunch & Bound and Cutting planes methods) with a meta-heuristic algorithm based on neighborhood structure.

2. PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

2.1 Problem definition and assumptions

In this section, a formulation of the budget-constrained DUFLNDP is proposed. First, the problem is formulated as a mixed-integer nonlinear programming model. In this model, some of the constraint sets have quadratic terms, but an equal mixed-integer linear model is presented by the definition of additional variables and also some needed constrains.

In addition to some assumptions that were considered for UFLNDP by Daskin et al. (1993), the following assumptions are also considered: (1) the facilities and links are uncapacitated, (2) parameters change over time with specific process, (3) once a link is built, it remains open throughout the time horizon but opened facilities may be closed in the subsequent periods, (4) opening and closing of facilities as well as constructing of links are instantaneous, (5) opening of a facility must happen at the beginning of a time period and such a facility may be closed in the future; (6) closing of a facility must happen at the end time period.

2.2 Notations

The model of the budget-constrained DUFLNDP is formulated below as a mixed-integer non-linear programming (MINLP) problem. The following notations are considered to model of the proposed problem.

Table 1: notations used in the proposed model

Symbol	Description
Sets	
N	set of network nodes (i,j) and clients k, $i, j, k \in \{1, 2, ..., \|N\|\}$,
NB	Set of opened facilities in existing network, $NB \in \{1, 2, ..., \|N\|\}$
LE^t	Set of existing links at time period t, $(i, j) \in LE^t$

LP^t	Set of potential links at time period t, $(i,j) \in LP^t$
L^t	Set of network links at time period t, $(i,j) \in L^t, L^t = LE^t \cup LP^t$
LB	Set of constructed links in existing network, $(i,j) \in LB$
T	Set of time periods, $t \in \{1,2,...,\lvert T \rvert\}$
Parameters	
d_k^t	The demand of client k at time period t;
LL_{ij}	The length of link (i,j);
P^t	The number of opened facilities at time period t;
gf_i^t	Fixed cost of opening a facility on node i at time period t which isn't
pf_i^t	Fixed cost of closing a facility on node i at time period t which is
gc_{ij}^t	Fixed cost of constructing a link (i,j) at time period t which isn't
tr_{ij}^t	Travel cost per unit flow on link (i,j) at time period t;
tr_{ij}^{kt}	The cost of travelling on link (i,j) if all the demand of client k goes
f_i^t	Operating cost of opened facility on node i during time period t;
c_{ij}^t	Operating cost of constructed link on (i,j) during time period t;
BF^t	Budget constrained for facilities at time period t;
BL^t	Budget constrained for network links at time period t;

Also, the decision variables of the model are defined as follows:

$Z_i^t = 1$ if facility i is open at the beginning of time period t else equal 0,

$X_{ij}^t = 1$ if link (i,j) is open at the beginning of time period t else equal 0,

Y_{ij}^{kt} Fraction of client's demand k traveling i to j at time period t,

W_i^{kt} Fraction of client's demand k served by facility i at time period t;

2.3 Model formulation

Using these notations and assumptions, the mathematical formulation of DUFLNDP is shown below:

Min

$$\sum_{t\in T}\sum_{(i,j)\in L^t}\sum_{k\in N} tr_{ij}^{kt} Y_{ij}^{kt} + \sum_{t\in T}\sum_{i\in N} f_i^t Z_i^t + \sum_{t\in T}\sum_{(i,j)\in L^t} c_{ij}^t X_{ij}^t \quad \textbf{(1)}$$

Subject to

$$Z_i^t + \sum_{j\in N} Y_{ij}^{it} = 1; \qquad ,\forall i \in N, t \in T \quad \textbf{(2)}$$

$$\sum_{j \in N} Y_{ji}^{kt} = \sum_{j \in N} Y_{ij}^{kt} + W_i^{kt}; \qquad , \forall i, k \in N : i \neq k, \forall t \in T \tag{3}$$

$$Z_k^t + \sum_{i \in N : i \neq k} W_i^{kt} = 1; \qquad , \forall k \in N, t \in T \tag{4}$$

$$Y_{ij}^{kt} \leq X_{ij}^t; \qquad , \forall (i,j) \in L^t, k \in N, t \in T \tag{5}$$

$$W_i^{kt} \leq Z_i^t; \qquad , \forall i, k \in N : i \neq k, t \in T \tag{6}$$

$$X_{ij}^t (1 - X_{ij}^{t-1}) + X_{ji}^t (1 - X_{ji}^{t-1}) \leq 1; \qquad , \forall (i,j) \in L^t, t \in T, \tag{7}$$

$$\sum_{i \in N} gf_i^t Z_i^t (1 - Z_i^{t-1}) + \sum_{i \in N} pf_i^t Z_i^{t-1} (1 - Z_i^t) + \sum gc_{ij}^t X_{ij}^t (1 - X_{ij}^{t-1}) \leq \beta^t, \qquad , \forall t \in T \tag{8}$$

$$X_{ij}^{t+1} \geq X_{ij}^t; \qquad , \forall (i,j) \in L^t, t \in T, t < T \tag{9}$$

$$Y_{ij}^{kt} \geq 0, \qquad , \forall (i,j) \in L^t, k \in N, t \in T \tag{10}$$

$$X_{ij}^t \in \{0,1\}, \qquad , \forall (i,j) \in L^t, t \in T \tag{11}$$

$$W_i^{kt} \geq 0, \qquad , \forall i, k \in N : k \neq i, t \in T \tag{12}$$

$$Z_i^t \in \{0,1\}, \qquad , \forall i \in N, t \in T \tag{13}$$

The objective function (1) includes the total cost over the time horizon and minimizes only the traveling and operating costs. These costs are composed of the two main components as sum of transportation costs and the operating costs of the facilities and of the network. However, investment costs are not considered in the objective function. Equations (2-4) are the flow conservation conditions, which must hold for each client, facility and period. Constraint (2) ensures that demand at i is either served by a facility at i or by shipping on some link out of i . Once a link is constructed, it remains open throughout the time horizon. Constraint (3) states in time period t and for client k that the flow in to i must equal the flow out of i . Constraint (4) imposes that in time period t and for client k, the demand must find a destination, whether it be at node k itself (z_k^t) or at the other nodes i (W_i^{kt}). Constraints (5) and (6) guarantee that in each time period, potential links and facilities are not used if they are not constructed.

Constraint (7) is equivalent to ones in UFLNDP that says on any given link, an optimal solution flow for a given time period will be in only one direction. Therefore, the links (i,j) and (j,i) cannot both be constructed in the static case.

However, in the dynamic case, once a link is constructed, it remains open throughout the time horizon. Therefore, at some time periods, the passing flow on some constructed links may be zero. This situation occurs when these links were built previously. If none of (i, j) and (j, i) links were constructed previously, one of them could be established at the current time period. In the other cases, at least one of the (i, j) and (j, i) links was established before. Thus, the other one (in diverse direction) can be constructed in the current time period.

The budget constraints represent by (8). Constraint (9) implies that once a link is constructed, it remains open throughout the time horizon. Constraints (10) and (12) enforce the non-negativity of the flow variables. Constraints (11) and (13) enforce the binary restriction on the location and link decision variables.

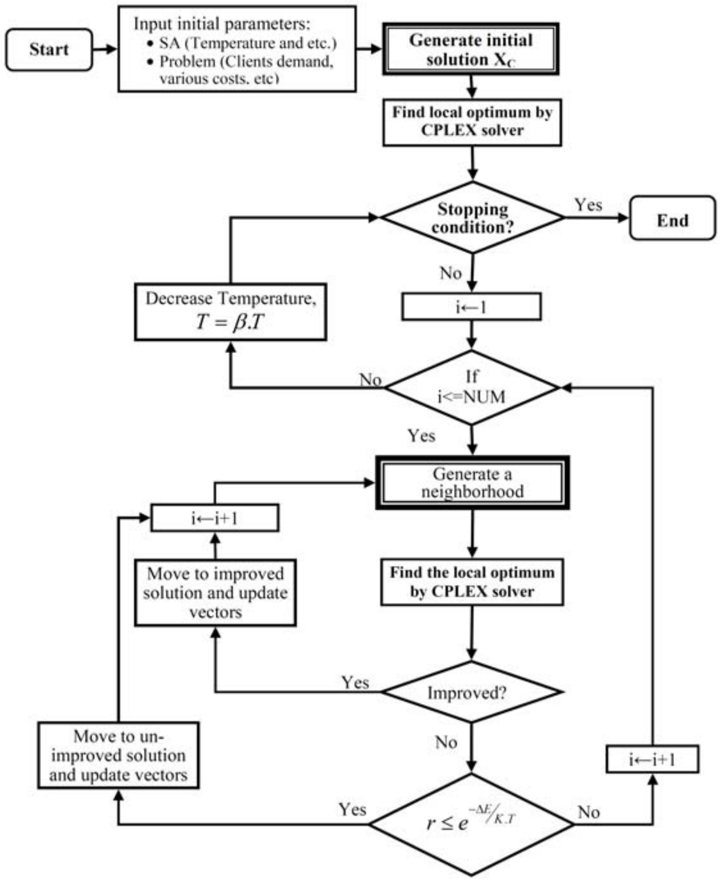
The budget-constrained DUFLNDP is a MINLP model because the proposed model has non-linear terms in some constraints. However, it can be easily linearized by introducing new binary variables and additional constraints.

3. SOLUTION PROCEDURES

The proposed model was coded by GAMS and solved by CPLEX solver. The performance of CPLEX is not very good to solve problems in proposed model. In the model, even finding a feasible solution for large scale instances is challenging. Thus, a hybrid heuristic is applied to solve the model. The proposed heuristic is an iterative hybrid metaheuristic algorithm, which directly based on the simulated annealing and the modified linear mixed-integer program. Ideas from the well-known SA methodology were used to avoid becoming stuck at a local optimum solution. In addition, an exact optimization algorithm that uses Branch & Bound and cutting plane methods in its framework was used to find an optimal solution.

Simulated annealing is a relatively old and effective metaheuristic aimed at solving combinatorial and global optimization problems. This algorithm is a local search that escapes from the local optima. The idea behind simulated annealing comes from the physical process of annealing, in which a solid is heated to a given temperature and then slowly cooled to achieve an optimal crystal structure. SA algorithm first appears in the literature by Kirkpatrick et al. (1983) and since then developed both in its methods and its applications. The overall flowchart of the hybrid heuristic is shown in Figure 1 and comprised of two main processes as SA and CPLEX solver operators.

Figure 1: The overall flowchart of the hybrid heuristic



Hence, the SA is applied to search on fixed variables that are Z_i^t. Then, the optimal solution of the subproblem is obtained by CPLEX solver. Finally, the heuristic terminates and reports the best found solution. There are some parameters in the proposed framework of hybrid heuristic as follows:

- IT: Initial Temperature
- T: The current Temperature
- NUM: The number of searched neighborhood at each temperature
- β: the cooling rate
- ΔE : the objective function difference between the current solution and its neighborhood
- K : a constant

- i: an counter

4. COMPUTATIONAL EXPERIENCE

In this section, a computational study was performed, and the performance of the proposed algorithm was tested. The mixed-integer programming model described in the previous sections was solved with standard mathematical programming software GAMS 23.3.3 by using the CPLEX 12.1 solver on a range of test problems. The proposed algorithm was coded in MATLAB. Software that makes an interface between MATLAB and GAMS was used. In addition, all programs were implemented on a Pentium IV PC with a 2.4 GHz processor and 2 GB RAM.

The dimensions of the test problems vary with a number of network nodes from 5 to 80 and time periods from 5 to 20. The results obtained show that the performance of CPLEX is different for these test problems. Thus, it can optimally solve the small instances in a reasonable execution time. It is also able to solve larger test problems but with much more CPU time. Furthermore, CPLEX does not have the memory required to handle large instances. As a result, the time needed to obtain an optimal solution of the proposed model by using CPLEX is very dependent on the dimensions of the problem.

4.2 Computational Results

Ten test problems with various dimensions were solved by the proposed heuristic and the CPLEX solver to construct a new network by the Model. The results obtained are shown in Table 2. The proposed hybrid heuristic results are compared with the lower and upper bounds obtained by CPLEX solver. The GAP of CPLEX solver and the hybrid algorithm is computed as follows. $Obj.$ is the value of the optimum or the best found solution, and LB is the lower bound reported by CPLEX after the given time limit.

$$GAP = \frac{Obj. - LB}{LB} * 100 \qquad \textbf{(20)}$$

The criterion for the termination of the proposed algorithm and CPLEX is one of the following conditions:

(1) A specified time limit has been reached (set as 60*N*T).
(2) The gap between the lower bound and upper bound is zero.

In order to implement the proposed algorithm efficiently, a sensitivity analysis of fix-and-optimize parameters is first conducted. First, twenty solutions was generated and sorted subject to the value of their objection function increasingly. The initial temperature was set as (0.01*OF1-0.001*(OF20-OF1). OF1 and OF20 are the first and twentieth sorted value of the objective function for generated solutions. The temperature updating scheme is by $T_{i+1} = 0.95 * T_i$. The number of iteration in each temperature is *NUM=10*. The value of *K=3* was chosen for instances with *N* equal to 40 or smaller and *K=1* for other instances.

As expected, the small test problems are solved optimally by both CPLEX and hybrid algorithm. CPLEX loses its performance with increasing in the problem dimensions such that it cannot even achieve a feasible solution for the last three instances with given stopping time. However, the hybrid heuristic find a feasible solution for all test instances in a few seconds. Furthermore, almost for all instances fix-and-optimize heuristic outperforms CPLEX and leads to better solutions in term of solution quality and CPU time. In the last two columns, the GAP and CPU time ratio of the proposed heuristic vs. CPLEX is demonstrated for all the solved test problems. These results reveal that the CPU time ratio is less than one for all test problems except for the two first ones. Thus, CPLEX only has better performance than the hybrid SA for these small instances. Taking all instances together, the difference average deviation of the solution obtained by the hybrid algorithm from the upper bound by CPLEX with the lower bound is almost 2.24% (12-9.76) (last row).

Figure 2: the number of test problems parameters (new networks) and computational results of CPLEX and the hybrid SA

	# of parameters			CPLEX				Fix-and-optimize heuristic		
Test Problems	**N**	**L**	**T**	**Obj.**	**CPU time (s)**	**Gap (%)**	**LB**	**Obj.**	**Bcpu (%)**	**Gap (%)**
TP1	5	18	5	27503.945	2	0.00	27503.945	27503.945	3.12	0.00
TP2	10	44	5	35359.026	3	0.00	35359.026	35359.026	5.89	0.00
TP3	10	38	10	153848.702	768	0.00	153848.702	153848.70	15.21	0.00
TP4	20	122	10	164566.559	12000	31.04	125580.811	158671.15	847.35	26.35
TP5	60	410	5	195377.498	18000	15.10	169751.996	190792.04	1881.51	12.39
TP6	60	360	10	413201.012	36000	20.74	342218.313	402615.27	15750.1	17.65
TP7	80	342	5	214529.538	24000	17.15	183122.707	205075.00	29873.8	11.99
TP8	20	122	30	out of	36000	-	238762.6216	291464.23	9150.07	22.07
TP9	40	324	20	NA	48000	-	457002.8	542758.30	34861.2	18.76
TP10	80	452	10	NA	48000	-	477002.2	551225.32	47620.7	15.56
Av.					**22277**	**12.00**			**7918**	**9.76***

***:** without the last three instances

Based on computational results, CPLEX is not very efficient to solve the budget-constrained UFLNDP in the large-scale instances. However, the proposed hybrid SA algorithm achieves significantly better solutions in comparison with CPLEX in almost all of test problems. Even for the smaller classes, the average execution time using CPLEX was higher than the proposed algorithm.

5. CONCLUDING REMARKS AND FUTURE DIRECTIONS

In this paper, a mixed-integer linear programming model was presented for the dynamic budget-constrained uncapacitated facility location-network design problem. In the second part of the paper, an efficient hybrid algorithm was proposed to solve the model. The simulated annealing algorithm and exact methods (Branch & Bound and Cutting methods) were combined to form this hybrid algorithm. The performance of the hybrid algorithm was compared with that of the CPLEX solver in the numerical experiments. The results showed that hybrid SA produces much better results than CPLEX for many test problems.

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